Solutions

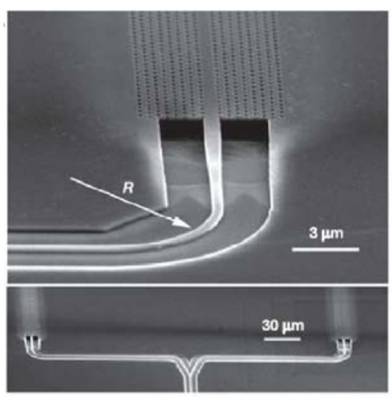
Problem 1 *(4.0 points)*

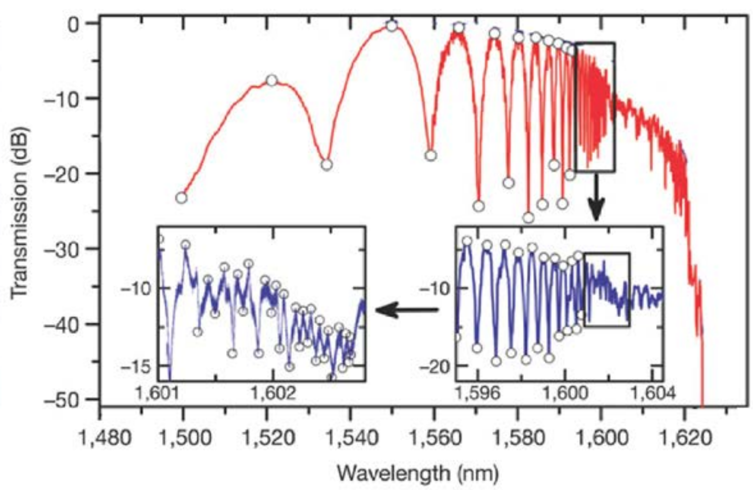
Slow light in photonic crystal waveguide (*Y. A. Vlasov, et al, Active control of slow light on a chip with photonic crystal waveguides, Nature 438, 65 (2005)*)

"*It is known that light can be slowed down in dispersive materials near resonances. Dramatic reduction of the light group velocity—and even bringing light pulses to a complete halt—has been demonstrated recently in various atomic and solid state systems… Here we experimentally demonstrate an over 300-fold reduction of the group velocity on a silicon chip via an ultra-compact photonic integrated circuit using low-loss silicon photonic crystal waveguides…*"

In the experiment, the authors fabricated an unbalanced Mach-Zehnder interformeter (MZI) having a reference arm and a signal arm. In both arms, there are photonic crystal waveguides (left figure) with length of 250 μm. Within the spectral range of interest (between 1,500 nm and 1,600 nm), the group index in the reference arm can be considered to be constant $n_g^{\mathrm{ref}} = 5$. The group index in the signal arm, however, varies strongly with the wavelength. Therefore, in the measured transmission spectrum of the MZI, strong interference fringes are observed, as shown in the curve in the right figure.

Task: Read data approxmately from the plot in the right figure to calculate the group index of the photonic crystal waveguide in the signal arm $n_g^{\mathrm{sig}}(\lambda)$ at three representative values of wavelength λ. Clearly write down the calculation method and the numerical values used to calculate. Try to find out the highest value of group index and give the corresponding group velocity of light.





Solution:

Group index in a dispersive media or photonic structures is given by:

$$n_g(\lambda) = n(\lambda) - \lambda \frac{dn(\lambda)}{d\lambda}$$

where *n(λ)* is refractive index or mode index.

The phase difference of the light wave after propagating in the two arms of the MZI is:

$$\Delta\phi = 2\pi\left[n^{\mathrm{sig}}(\lambda) - n^{\mathrm{ref}}(\lambda)\right]L/\lambda$$

The wavelengths of the neighboring maxima and minima in the interference fringe are λ_{max} and λ_{min}, respectively. They are given by:

$$\Delta\phi(\lambda_{\mathrm{max}}) = 2\pi\left[n^{\mathrm{sig}}(\lambda_{\mathrm{max}}) - n^{\mathrm{ref}}(\lambda)\right]L/\lambda_{\mathrm{max}} = 2N\pi$$

$$\Delta\phi(\lambda_{\mathrm{min}}) = 2\pi\left[n^{\mathrm{sig}}(\lambda_{\mathrm{min}}) - n^{\mathrm{ref}}(\lambda)\right]L/\lambda_{\mathrm{min}} = (2N-1)\pi$$

where N is an integer. From above we can get:

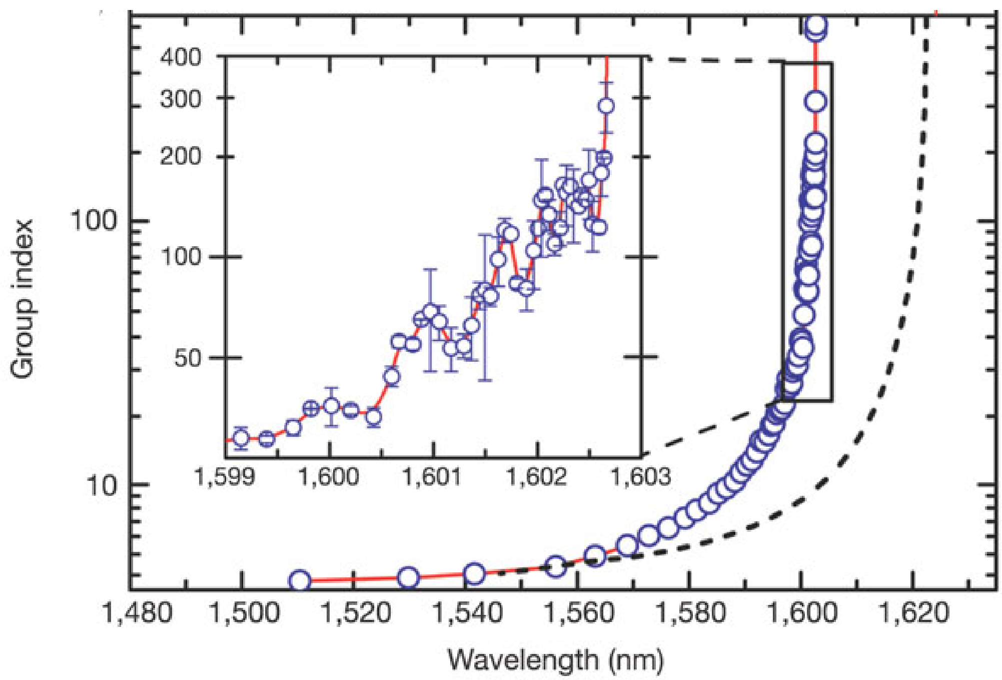
$$n^{\mathrm{sig}}(\lambda_{\mathrm{max}}) = N\frac{\lambda_{\mathrm{max}}}{L} + n^{\mathrm{ref}}(\lambda_{\mathrm{max}}) \text{ , } n^{\mathrm{sig}}(\lambda_{\mathrm{min}}) = (N-\frac{1}{2})\frac{\lambda_{\mathrm{min}}}{L} + n^{\mathrm{ref}}(\lambda_{\mathrm{min}})$$

So the group index at λ_{max} can be expressed by:

$$\begin{aligned}
&n_{\mathrm{g}}^{\mathrm{sig}}(\lambda_{\mathrm{max}}) = n^{\mathrm{sig}}(\lambda_{\mathrm{max}}) - \lambda_{\mathrm{max}}\left.\frac{dn(\lambda_{\mathrm{max}})}{d\lambda}\right|_{\lambda_{\mathrm{max}}} \\
&\approx N\frac{\lambda_{\mathrm{max}}}{L} + n^{\mathrm{ref}}(\lambda_{\mathrm{max}}) - \lambda_{\mathrm{max}}\frac{n(\lambda_{\mathrm{max}}) - n(\lambda_{\mathrm{min}})}{\lambda_{\mathrm{max}} - \lambda_{\mathrm{min}}} \\
&= N\frac{\lambda_{\mathrm{max}}}{L} + n^{\mathrm{ref}}(\lambda_{\mathrm{max}}) - \lambda_{\mathrm{max}}\frac{\left[N\frac{\lambda_{\mathrm{max}}}{L} + n^{\mathrm{ref}}(\lambda_{\mathrm{max}}) - (N-\frac{1}{2})\frac{\lambda_{\mathrm{min}}}{L} - n^{\mathrm{ref}}(\lambda_{\mathrm{min}})\right]}{\lambda_{\mathrm{max}} - \lambda_{\mathrm{min}}} \\
&= N\frac{\lambda_{\mathrm{max}}}{L} + n^{\mathrm{ref}}(\lambda_{\mathrm{max}}) - \lambda_{\mathrm{max}}\frac{\left[N\frac{\lambda_{\mathrm{max}} - \lambda_{\mathrm{min}}}{L} + \frac{\lambda_{\mathrm{min}}}{2L} + n^{\mathrm{ref}}(\lambda_{\mathrm{max}}) - n^{\mathrm{ref}}(\lambda_{\mathrm{min}})\right]}{\lambda_{\mathrm{max}} - \lambda_{\mathrm{min}}} \\
&= \frac{\lambda_{\mathrm{max}}\lambda_{\mathrm{min}}}{2L(\lambda_{\mathrm{max}} - \lambda_{\mathrm{min}})} + \left\{n^{\mathrm{ref}}(\lambda_{\mathrm{max}}) - \lambda_{\mathrm{max}}\frac{n^{\mathrm{ref}}(\lambda_{\mathrm{max}}) - n^{\mathrm{ref}}(\lambda_{\mathrm{min}})}{\lambda_{\mathrm{max}} - \lambda_{\mathrm{min}}}\right\} \\
&\approx \frac{\lambda_{\mathrm{max}}\lambda_{\mathrm{min}}}{2L(\lambda_{\mathrm{min}} - \lambda_{\mathrm{max}})} + n_{\mathrm{g}}^{\mathrm{ref}}
\end{aligned}$$

where approximations assume $\lambda_{\mathrm{max}} - \lambda_{\mathrm{min}} \ll \lambda_{\mathrm{max}}, \lambda_{\mathrm{min}}$.

Therefore the values of λ_{max} and λ_{min} can be read from the plots and the group index and group velocity $v_g = c/n_g$ can be determined as in the following plot.



400
300
200
100
50
1,599
1,600
1,601
1,602
1,603
100
10
Group index
1,480
1,500
1,520
1,540
1,560
1,580
1,600
1,620
Wavelength (nm)